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Equivalent elastoplastic constitutive spring model for the seam of stand seam roofs

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SUMMARY:
An equivalent elastoplastic constitutive spring model is formulated to describe the irreversible deformation and coupling relationship between the seams and the clips of stand seam roofs. This paper presents an approach for calibrating and validating the constitutive model via component tests of stand seam roofs. Firstly, according to the uniaxial pull-out test results in three directions, the optimal solutions of the parameters in the constitutive model are calibrated by the least squares method. Subsequently, the constitutive model is validated by considering its performance in pull-off tests in both directions. The results demonstrate that the equivalent elastoplastic spring model can well describe the response characteristics of multiaxial stress state of stand seam roofs.

Keywords: Constitutive model, Elastoplastic, Spring, Calibration, Stand seam roofs

1. GENERAL INSTRUCTIONS

Stand seam roofs have been widely used on buildings because of its excellent bearing capacity, good thermal insulation and waterproof performance (Krivoshapko,2017). However, the failure of the stand seam roofs under wind load often occurs, which brings severe challenges to the use and maintenance of buildings. Post-disaster investigation and wind resistance test studies showed the clips and seam connection are easy to break in the wind-induced destruction (Lu, 2022). To facilitate engineering applications and accurately predict the safety of stand seam roofs, a sophisticated numerical model can help to describe the mechanical behaviours of this kind of structure.

The numerical model to describe the nonlinear mechanical behaviour of the seam can be generally at least divided into two categories: contact numerical models and equivalent constitutive spring model. The contact numerical models can simulate the whole process of roof seam failure (Ali and Paul,2003). However, the contact theory is too complex for engineering applications, and the calculation is not easy to converge. The equivalent constitutive spring model has simple form and high precision when simulating the stress state of stand seam roofs (El Damatty et al., 2003). However, currently only the linear elasticity phase of the seam can be simulated by equivalent constitutive spring model. When the system has plastic deformation, there is a coupling effect between the equivalent constitutive spring model stiffness in different directions, and the spring stiffness will change with the plastic deformation. Considering the coupling effect of the spring stiffness, this paper proposes an equivalent elastoplastic spring model, which can be used to simulate the nonlinear response characteristics of the stand seam roof of under wind pressure.

2. CONSTITUTIVE SPRING MODEL OF STAND SEAM ROOF

The equivalent constitutive spring model of stand seam roof simplifies the connection of the seam with the clips to an equivalent spring system, as shown in Figure 1. In the Figure, u_1 and u_2 are the degrees of freedom in the x and y direction of the spring which simulates the horizontal and vertical support provided by the seam within the roof, u_3 is the degree of rotational freedom of the spring around the z direction which simulates the bending stiffness provided by the seam around the longitudinal axis of the roof. f_1, f_2, f_3 is the spring vertical, horizontal internal force and bending moment respectively.

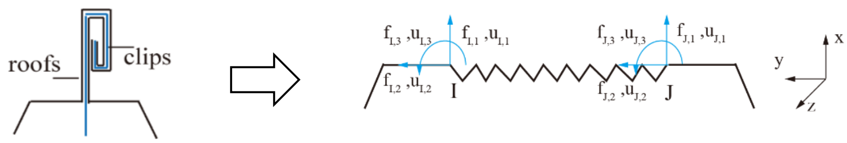


Figure 1. The degrees of freedom of the elastoplastic spring model(El Damatty et al, 2003).

The elastoplastic equivalent spring displacement increment du is composed of two parts: elastic relative displacement increments du^e and plastic relative displacement increment du^p.When unloaded to zero, the plastic displacement still exists, indicating that the spring internal force increment $d\boldsymbol{f}$ is only related to the elastic relative displacement increment as follows:

$$d\boldsymbol{f} = \boldsymbol{K_S} du^e = \boldsymbol{K_S}(du - du^p) \quad (1)$$

where $\boldsymbol{K_S}$ is the elastic stiffness matrix of the spring and the diagonal matrix formed by K_v, K_h, K_θ. And $K_v. K_h, K_\theta$ are vertical, horizontal and bending spring stiffness, respectively.

The internal force of the spring is related to the equivalent plastic relative displacement of the spring $\bar{u}_p$, which is defined as:

$$\bar{u}_p = \sqrt{u_i^p u_i^p} \quad (i=1,2,3) \quad (2)$$

where u_i^p is a tensor representing plastic relative displacement in the three degrees of freedom of the spring, and $\bar{u}_p$ is the accumulation of plastic relative displacement. For the elastoplastic equivalent spring model, the yield function is:

$$f_p(\boldsymbol{f}, \bar{u}_p) = \sqrt{f_1^2 + (b_1 f_2)^{\gamma_1} + (b_2 f_3)^{\gamma_2}} - [a_0 + a_1(1 - e^{-m\bar{u}_p})] \quad (3)$$

where a_0 is the initial yield surface. a_1 is the maximum value of the subsequent yield surface, and m is the rate of change of yield surface size with plastic deformation. b_1, γ_1, b_2, γ_2 are spring material parameters. These parameters need to be calibrated by experiments.

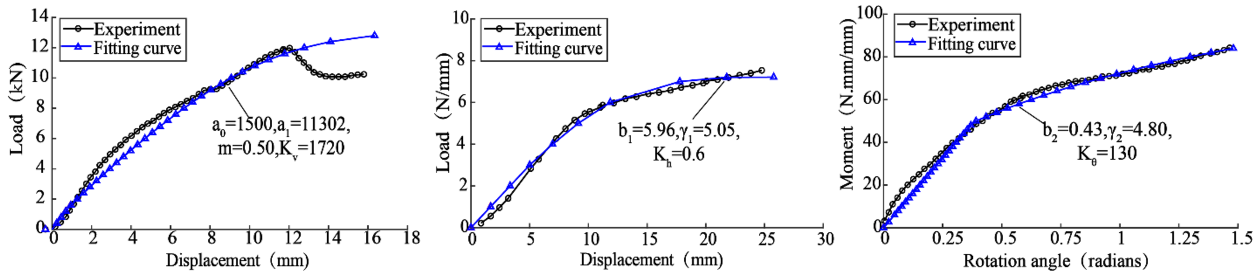
The flow rule chosen was associative. Therefore, the plastic relative displacement rate tensor du^p is defined as follows:

$$du^p = \dot{\lambda}_p \frac{\partial f_p}{\partial \boldsymbol{f}} \quad (4)$$

where $\dot{\lambda}_p$ is the plastic multiplier, which can be solved by the consistency condition.

3.CALIBRATION OF THE PARAMETERS

El Damatty et al. (2003) derived three sets of specimens containing the seams and clips from the stand seam roof and performed uniaxial pull-out tests to determine the stiffness of the equivalent spring model. The typical load displacement curve of the test specimen is shown in Figure 2.



(a)Vertical fitting curve (b) Horizontal fitting curve (c) Bending fitting curve

Figure 2. The fitting curve and experimental curve of equivalent constitutive spring model

The parameters that need to be calibrated are concentrated in the yield function, and a total of 10 parameters need to be calibrated. The parameters can be divided into three groups according to their constitutive meaning. (1) The parameters a_0, a_1, m and K_v are related to the vertical bearing capacity of the seams, which can be determined by the vertical pull-out tests; (2) The parameters b_1, γ_1, K_h control the horizontal displacement and internal force of the seams, which are calibrated by the horizontal pull-out tests.(3) The parameters b_2, γ_2, K_θ control the bending resistance of the seams, and the parameters are calibrated by bending tests. The least squares method is used in this paper to fit the above three groups of parameters. The fitting results of the three groups of parameters are shown in Figure 2, and finally the specific values of the ten parameters are shown in Table 1.

Table 1. The value of the parameters.

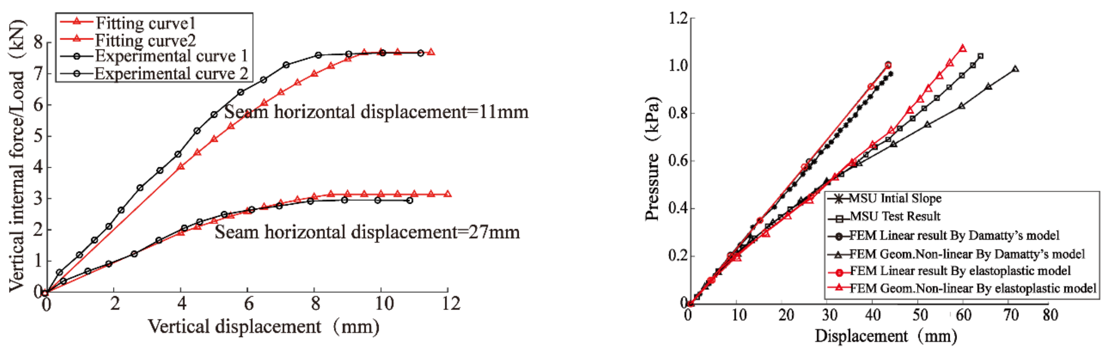
Parameters	a_0	a_1	m	b_1	b_2	γ_1	γ_2	K_v	K_h	K_θ
Value	1500	11302	0.50	5.96	0.43	5.05	4.80	1720	0.60	130

4.VALIDATION OF THE CONSTITUTIVE MODEL

This paper verifies the accuracy of the equivalent elastoplastic spring model based on the experiment carried out by El Damatty and Rahman (2004) to study the relationship between the clip stiffness and the seam deformation. Fig. 3(a) shows the relation curve between vertical displacement u_1 and load f_1 of the clips under different horizontal displacement u_2 of the seams obtained in the experiment. The relationship between the vertical internal force and displacement of the spring shows a nonlinear behavior, which will be theoretically explained by the equivalent elastoplastic constitutive spring equation, Eq.(4).

The constitutive equation is obviously a nonlinear equation. The MATLAB is used to solve the nonlinear equation to obtain the corresponding spring vertical internal force f_1 to the displacement value u_1 under different seam horizontal displacements u_2. The relationship is shown in Fig. 3(a). It is basically the same as the curve obtained by the experimental curve, the error is less than 10%. The ultimate bearing capacity, that is, when the curve tends to be horizontal, of the experimental and theoretical curves is equal. The equivalent elastoplastic model of spring is proved to be correct.

A detailed finite element model based on ABAQUS was developed for analyzing the wind resistance of stand seam roofs. Comparing with the MSU test (El Damatty et al., 2003), the responses of the roofs under the elastic equivalent model proposed by El Damatty et al. (2003) and the equivalent elastoplastic constitutive spring model proposed in this paper are calculated, the results are shown in Fig. 3(b). It can be seen that the equivalent elastoplastic constitutive spring model proposed in this paper is closer to the test results, and the nonlinear behavior of the roof is simulated better.



(a)Vertical force-displacement curve under different horizontal displacements (b)Pressure-displacement curve

Figure 3. Comparison of numerical and experimental results

5. CONCLUSIONS

The equivalent elastoplastic spring model of seams is proposed in this paper, and the unknown parameters of the constitutive in the equivalent spring model are calibrated by fitting the load displacement curves of three sets of uniaxial pull-out experiments in different directions, then the correctness of constitutive parameters and equivalent models is verified through pull-out experiments in two directions. A theoretical explanation is given that the stiffness of the clips decreases with the increase of the horizontal displacement of the seams. The equivalent elastoplastic spring model proposed in this paper can simulate the nonlinear behavior of the seams and clips in the stand seam roof, which can be used for the numerical analysis of the stand seam roof to calculate the ultimate bearing capacity under multiaxial loading and the displacement response to the cyclic wind load.

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